

## Representation and the Computation of Tone Processes

**Main Result:** Enhancing the representation provides a tight characterization of the computationally complex tone processes noted in (Jardine, 2016)

- *Unbounded circumambient* (UC) processes, are complex tone processes in which triggers or blockers can be arbitrarily far away on either sides of any target.
- E.g: Unbounded Tone Plateauing (UTP) in Luganda (Hyman and Katamba, 2010) is a case of UC, H(igh) tones on either side of an unbounded span of toneless TBUs form a single H-toned plateau as in (1).

(1) tw - áa - láb - w - a wal ú simbi → tw - áá - láb - w - á wál ú simbi ‘We saw him, Walusimbi’

- Jardine argues that UC processes are not *subsequential*, a class of functions that can be computed over deterministic finite-state transducers (Mohri 1997).
- subsequentiality has been argued to form a tight bound on segmental phonology (Heinz and Lai 2013, Heinz 2018).
- We show that **extending a logical notion of subsequentiality to autosegmental representations (ARs; Goldsmith 1976) allows us to capture UC processes in tone**, thus providing a sufficiently expressive, yet restrictive characterization of tone.
- We use implicitly defined (Rogers, 1997) least-fixed point (lfp, Chandlee and Jardine 2019)-extended quantifier free (qf, Courcelle 1994) formulae.

**Cilungu:** (Bickmore, 1996) [unbounded H spread, Left-Subsequential]

$$(2) \quad a. \alpha'(x) \approx y \stackrel{\text{def}}{=} \underbrace{\alpha(x) \approx y}_{(i)} \vee \underbrace{\alpha'(p(x)) \approx y \wedge \neg \text{last}(x)}_{(ii)}$$

b.

(input) (output)

The definition in (2a) reads as follows:

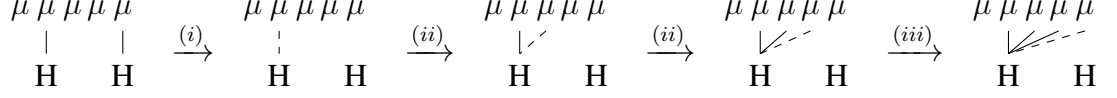
- The first conjunct  $\alpha(x) \approx y$  in (i) is true when  $x$  is associated to  $y$  ( $\alpha$  points  $x$  to  $y$ ) in the *input*, and the second conjunct  $\alpha'(p(x)) \approx y \wedge \neg \text{last}(x)$  in (ii) is true when the *predecessor* of  $x$  ( $p(x)$ ) is associated to  $y$  in the *output*, as long as  $x$  is non-final ( $\neg \text{last}(x)$ ).
- Thus, the definition states that  $\alpha'(x) \approx y$ —that is,  $x$  is associated to  $y$  in the output—if and only if  $x$  and  $y$  satisfy either disjunct (i) or (ii).
- A derivation is given in (2b) to show how successive pairs are associated, converging to an output in which H spreads up to the penult.

**Luganda:** (Hyman and Katamba, 2010) [UTP, non-subsequential]

$$(3) \quad \text{a. } R(x, y) \stackrel{\text{def}}{=} \underbrace{(\alpha(x) \approx y \wedge \text{first}(y))}_{(i)} \vee \underbrace{(R(p(x), y) \wedge \neg(\text{last}(\alpha(x))))}_{(ii)}$$

$$\text{b. } \alpha'(x) \approx y \stackrel{\text{def}}{=} R(x, y) \vee \underbrace{(\text{last}(\alpha(x)) \wedge \text{first}(y))}_{(iii)}$$

c.



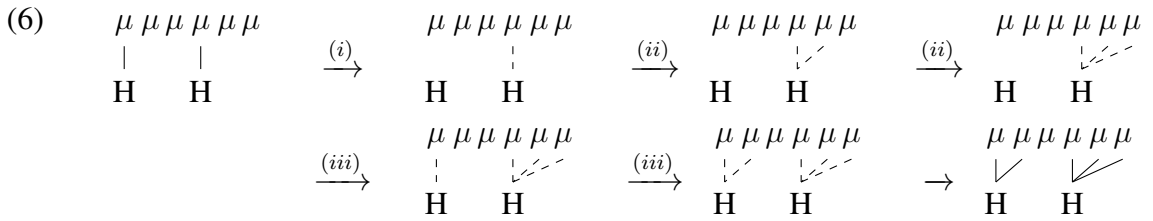
The labeled disjuncts work as follows:

- (i) states that  $x$  and  $y$  are associated if  $x$  is associated to the first  $y$  in the input; or if the predecessor of  $x$  on the timing tier is associated to  $y$  and it is not the case that the position to which  $x$  is associated is last (ii).
  - (iii) associates the TBU associated to the final tone ( $\text{last}(\alpha(x))$ ) to the first tone ( $\text{first}(y)$ ).
  - (3c) shows a step-by-step derivation of (3a-b). (We omit the deletion of the second H, but this is also QFLFP-definable).
- (4)

**Copperbelt Bemba** (Bickmore and Kula, 2015) [bounded and unbounded H spread, non-subsequential]

$$(5) \quad \text{a. } R(x, y) \stackrel{\text{def}}{=} \underbrace{(\alpha(x) \approx y \wedge \text{last}(y))}_{(i)} \vee \underbrace{(R(p(x), y))}_{(ii)}$$

$$\text{b. } \alpha'(x) \approx y \stackrel{\text{def}}{=} R(x, y) \vee \underbrace{((\alpha(x) \approx y \vee \alpha(p(x)) \approx y \vee \alpha(p(p(x))) \approx y) \approx y)}_{(iii)} \wedge \underbrace{\neg s(x) \approx s(y)}_{(iv)}$$



The formulae work as follows:

- Disjunct (i) first considers the pair that are associated in the input *only if*  $y$  is the last tone. The disjunct in (ii) then recursively associates subsequent TBUs, analogous to unbounded spreading in (2).
- The fact that this recursion only begins in (i) with the last tone captures the generalization in Bemba that only the last tone spreads unboundedly.
- The definition in (5b) then captures the bounded case: (iii) states that  $x$  is associated to  $y$  if they are associated in the input, *or* if  $p(x)$  is associated to  $y$  in the input (binary spreading), *or*  $p(p(x))$  is associated to  $y$  in the input (ternary spreading).

- (iii) is subject to the conjunction in (iv) (the OCP condition):  $x$  and  $y$  can only associate when there is no following input association (here,  $s(x)$  indicates the immediate *successor* of  $x$ ).

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